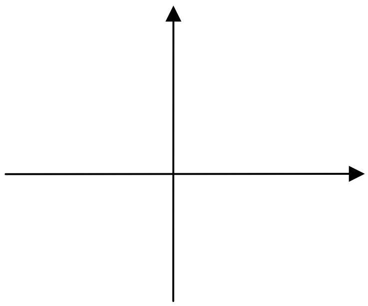
Math 253 – Fall 2008 **Test 3 (16.1 – 18.2)** **Name:______________________________**

You must show complete work for credit.

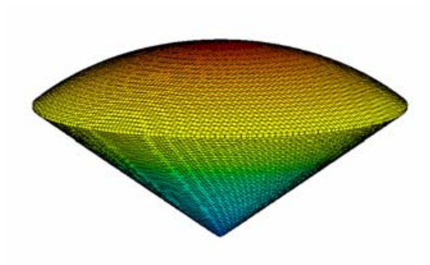
1) $\int_0^1 \int_y^1 \sin(x^2)\,dx\,dy$

a) Sketch the region of integration.



b) Reverse the order of integration and evaluate (leave your answer in *exact form*).

2) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} dz\, dy\, dx$ represents the volume of an "ice cream cone" shaped region bounded below by the cone $z = \sqrt{x^2 + y^2}$, and bounded above by a hemisphere $z = \sqrt{2 - x^2 - y^2}$.



a) Express this integral in cylindrical coordinates (do not evaluate).

b) Express this integral in spherical coordinates (do not evaluate).

3) A cork is thrown into a stream of water that has the velocity field $\mathbf{F} = y\,\mathbf{i} - x\,\mathbf{j}$.

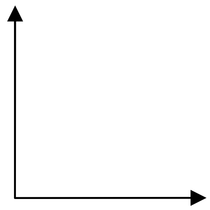
a) Set up a system of differential equations for the *flow line* the cork will follow.

b) Show that $x(t) = a\sin(t),\ y(t) = a\cos(t)$ is a solution to the system of differential equations.

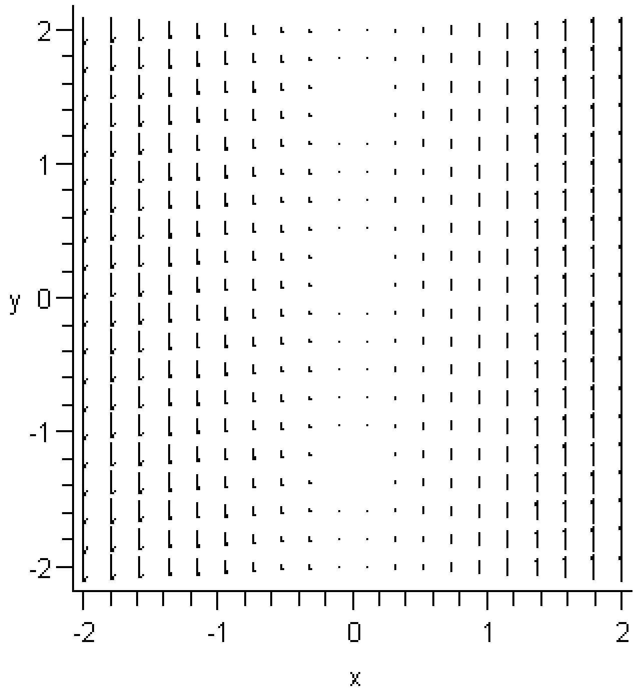
4) An airplane is flying in stormy weather. The velocity of the wind at each point (x,y,z) is given by $\mathbf{W}(x, y, z) = x^3\mathbf{i} + y^2\mathbf{j} + z\mathbf{k}$. Use the *method of parameterization* to calculate the amount of work the wind does on the plane as it flies in a straight line from the point (0, 2, 3) to the point (2, 5, 4).

5) Conceptual Problems:

a) Draw a triangular region that could be expressed as a *single* double-integral in the order dx dy, but would require more than one double-integral in the dy dx order.



b) The vector field **F** is shown below. Draw a path C such that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is clearly negative. Be sure to indicate the direction of the path!



Let R be the cube with one corner at (0, 0, 0) and its opposite corner at (2, 2, 2), and let the *density* at each point in the cube be given by a function $f(x, y, z)$.

c) Write a triple-integral representing the *mass* of the cube.

d) Write a triple-integral representing the *average density* of the cube.

e) Let R be the area inside the unit-circle centered at the origin. Determine the value of c that will make the integral zero. (Hint: there is no need for integration… think *graphically*).

$$\iint_R (2x + c)dA$$

C = _______